# Diagnostic Testing of Stochastic Circuits

G.J. Bertini
Novinium, Inc.
22019 70th AVE S
Kent, WA  98032 USA

***Abstract-* The desire of circuit owners to prioritize their underground cable rehabilitation expenditures has spawned a variety of diagnostic testing tools. The goal of these diagnostic tests is to separate the "at risk" cable population into the subsets of those that are likely to provide reliable service for some time and those that are likely to fail first – the "good" from the "bad." A decade of field failure data, covering over 18,000 km of cable installed over the 3 decades between 1969 and 1999 examined in this paper, provide statistically significant insights into the stochastic nature of cable reliability. A low cost method to prioritize circuit rehabilitation efforts is described. The method provides the foundation for the economic modeling and optimization of various rehabilitation strategies.**

## I. INTRODUCTION

If there were a cost-effective, non-destructive method to distinguish cables, which will fail in the near future from those that will provide some additional years of reliable service, every circuit owner in the world would utilize the method. No such method exists.

One North American circuit owner has gathered and shared in [1], over a decade of failure data on an installed base of over 29,000 km of XLPE (cross-linked polyethylene) cable. Fig. 1 shows the history of the cable installation. This paper will examine the largest of the three data sets representing 18,000 km, namely the 7-strand, 33.6 mm$^2$ (AWG No. 2) cable. This cable was unjacketed prior to 1985 and typically had a nominal insulation thickness of 4.5 mm.

The 33.6 mm$^2$ cable has a failure history provided in [1], which is reproduced in Fig. 2. In general the older the cable, the lower its service reliability, but there are some vintages that depart from that general trend. While the historical view of Fig. 2 is interesting, of even more value would be a prospective view, which provides statistically valid predictions of future failure rates.

## II. DATA

In addition to the data summarized in Fig. 1 and Fig. 2, one other kind of data is required to complete an analysis. Some portion of each cable vintage is removed or suspended from the population when the circuit owner employs a cable rehabilitation program. The suspension data considered in this paper include only cable replacement. Together these three data sets include a great deal of information: 35 years of installation data, 5 years of rehabilitation suspension data by installation year, and failure history for each vintage for 10 years.

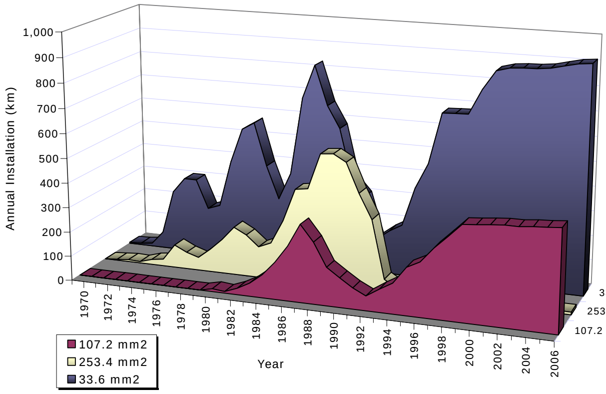

**Figure 1. Installation history for U.S. circuit owner.**

It is not possible in this paper to provide all of the raw data. To illustrate the analytical methods employed, this paper will examine three exemplary years, namely 1975, 1976 and 1977. The raw data are presented in Table 1 and Table 2. These methods will be extended to the entire data set.

## III. CROW-AMSAA

In [1], the author utilized a two-parameter Weibull model to fit the same failure data. In this paper the author analyzes the same data set with the Crow-AMSAA model (C-A) described by Eq. 1.

$$N(t) = \lambda t^{\beta} \qquad (1)$$

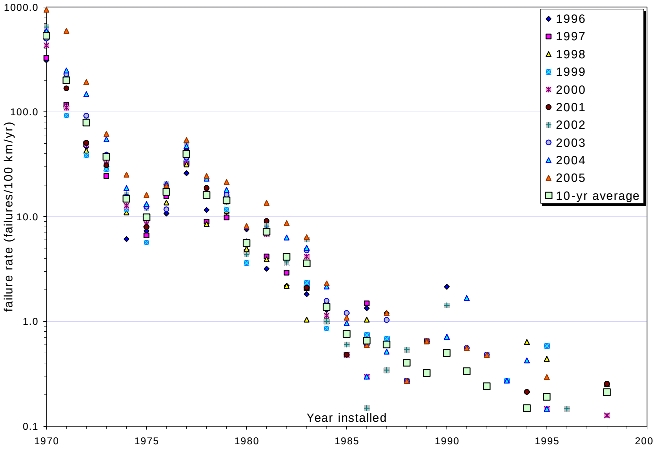

**Figure 2. Failure data of 30 years of vintage cable over a 10 year period and the 10-year simple average.**

**Table 1. Exemplary cable installation and suspension data for 1975–77. All values in kilometers.**

| | Vintage: | 1975 | 1976 | 1977 |
|---|---|---|---|---|
| | Installed | 423 | 205 | 216 |
| Suspended | 2001 | 32.2 | 2.1 | 0.0 |
| | 2002 | 19.4 | 2.8 | 2.5 |
| | 2003 | 26.2 | 4.1 | 2.7 |
| | 2004 | 11.0 | 2.1 | 0.4 |
| | 2005 | 25.1 | 2.1 | 0.2 |

C-A is a power curve of cumulative failures against the cumulative product of time-since-installation and length. The trend-line slope (beta) statistic provides insight into whether failures are increasing (β>1), decreasing (β<1), or remaining static (β=1). The y-intercept (lambda) is a hypothetical value, which allows the forecasting of future failures.

While the Weibull model used in [1] fits the data with roughly equal goodness-of-fit and statistical robustness (i.e. both have about the same correlation coefficient and utilize the same number of parameters), Weibull is more difficult to use in practice when there is an incomplete history of failures. C-A is theoretically less sensitive to multi-mode failure mechanisms as suggested by [2], however, the equivalent goodness of fit of the Weibull approach suggests that the bulk of failures are accounted for adequately with a single mode failure model. Except for the most modern vintages of cable, there are no early failure data available from the circuit owner. C-A would be expected to make a better prediction of future failures given the limitations of the available data.

## IV. GRANULARITY

There are two important issues of data granularity. The first is the choice of time increment. There is a pronounced seasonality of failures in non-tropical portions of the Northern and Southern Hemispheres. Failure rates peak during the late summer or early fall. Thus, periods of less than a year are statistically confounded with seasonal weather variations. Planning is usually done on an annual basis, so time measured in years is appropriate.

**Table 2. Failures over 10 years for 1975-77 vintage cables.**

| | Vintage: | 1975 | 1976 | 1977 |
|---|---|---|---|---|
| Calendar year failures | 1996 | 56 | 22 | 31 |
| | 1997 | 70 | 32 | 28 |
| | 1998 | 68 | 28 | 36 |
| | 1999 | 78 | 41 | 24 |
| | 2000 | 87 | 40 | 36 |
| | 2001 | 91 | 41 | 31 |
| | 2002 | 108 | 41 | 45 |
| | 2003 | 77 | 23 | 43 |
| | 2004 | 99 | 40 | 44 |
| | 2005 | 113 | 38 | 50 |

Not so obvious is the second granularity choice: What is the appropriate model length? If the circuit owner desires to model past and future reliability performance the choice must be made to accommodate any rehabilitation tactics. Potential tactics include:

1. Fix the failure with a repair length splice or two splices and a short length of approximately 30.5 cm (1 foot) of new cable.
2. Identify a sub-segment of the cable to be rehabilitated (i.e. chemically rejuvenated or replaced) based upon a diagnostic test. A sub-segment is a portion of a cable segment. A segment is delineated by two terminations
3. Rehabilitate entire cable segments.

The lowest common denominator of the three choices is the 30.5 cm, which generally must be replaced after a failure occurs.

## V. LEAST SQUARES

Fig. 3 provides a graphical view of the least squares fitting of the 1975 – 1977 data. The logarithmic scale of the x-axis represents the product of the "cumulative suspended" cable in meters and the years in service. The "cumulative suspended" cable is the cumulative installed cable less the suspended (rehabilitated) cable. Because portions of a cable population may be suspended by rehabilitation, x-values may decrease. The logarithmic scale of the y-axis measures the cumulative observed failures. There simply were no failures recorded prior to 1996, and hence the portion of the model prior to the 10 years of data is a backward extrapolation. Similarly, the portion of the model after the 10 years of data is a forward extrapolation, which assumes no cables from the three populations are rehabilitated after 2006.

The pronounced "lightning bolt" shape of the 1975 data is due to the circuit owner's aggressive rehabilitation of that vintage of cable between 2001 and 2005, as demonstrated in Table 1. Modest rehabilitation strategies for 1976 and 1977 vintage cables create barely visible jogs in those curves.

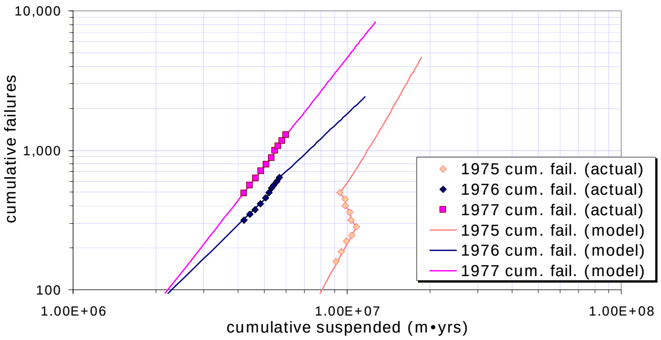

**Figure 3. 1975-77 data and C-A models. The least squares slope (β), intercept (λ), and correlation coefficient ($r^2$) are:**
**1975: 3.6, $2.19\text{x}10^{-25}$, 0.997;**
**1976: 1.9, $6.97\text{x}10^{-12}$, 0.996;**
**1977: 2.5, $6.00\text{x}10^{-16}$, 0.999.**

Between these three vintages of cable, the circuit owner should target the 1975 vintage first, then the 1977 vintage, and finally the 1976 vintage. It appears from a careful study of Table 1 that rehabilitation resources are being applied inappropriately to the 1976 vintage cable (β=1.9) ahead of the 1977 vintage cable (β=2.5). Considering just the 1975 – 1977 cables, a non-critical glance at Fig. 2 might lead the strategic planner to inappropriately apply resources to 1977 first, 1976 second and 1975 last, instead of 1975, 1977, and then 1976, as suggested by the beta statistics.

While soil conditions or landowner improvements in particular neighborhoods may create variability in rehabilitation costs, generally, the cost of rehabilitation is largely independent of vintage. If the circuit owner's goal were to provide the maximum overall reliability benefit for a given expenditure, rehabilitation resources should be applied to the worst performing cables first. In fact, the optimum strategic solution is to expend resources targeting 100% rehabilitation of the population of the worst performing cables as described by the hierarchy of needs provided in [1]. The worst performing cable population is that population with the highest failure rate. Fig. 4 provides a view of the C-A predicted failure rates for 2006 compiled for vintages spanning from 1970-85 alongside the rehabilitation intensity the circuit owner applied during the period from 2001-05.

The circuit owner has applied some rehabilitation budget in chemical rejuvenation, which is not reflected in Fig. 4. If one ignored the rejuvenation effort and considered only the replacement portion of the rehabilitation effort, it might appear that the circuit owner has applied its rehabilitation budget to the oldest cables first. Such a strategy might inappropriately under-emphasize 1977-79 vintage cables and overemphasize 1970-71 and 1974-76 vintage cables.

Not all failures are equal. Reliability spending may be driven by non-economic considerations and more refined reliability measurements than failure counting alone. There may be classes of failures that require attention sooner than indicated by Fig. 4. For example, a failure in a radial circuit will likely suffer an extended duration compared to a loop-fed circuit. While it is beyond the scope of this paper, many such additional considerations can be modeled, to the extent that the cable can be appropriately classified.

## VI. STOCHASTIC CIRCUITS

Once the C-A regression is performed on each population of cable vintages, history can be reconstituted and the future failures can be estimated for any rehabilitation scenario. Fig. 5 shows the entirety of the historical data classified for simplicity into six population groups. These groups are the pre-1975 vintage cables, 1976-80 vintage cables, 1981-85 vintage cables, 1986-90 vintage cables, 1991-2000 vintage cables and 21$^{st}$ century cables. In Fig. 5 it is assumed that all rehabilitation efforts are suspended in 2006. Consequently, the failure rates kink upward in 2007.

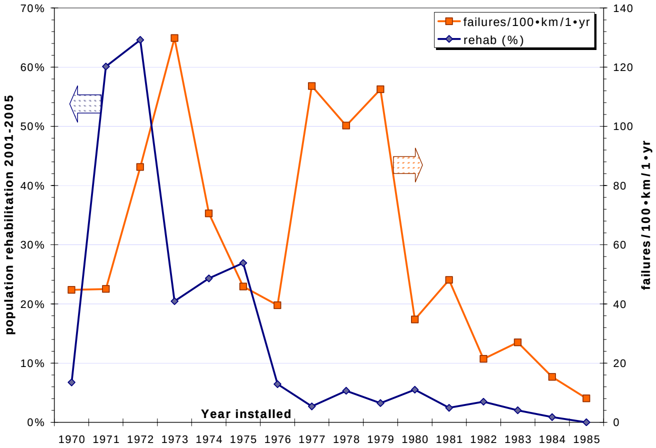

**Figure 4. Replacement of the original cable during 2001-05 is a reflection of strategy. Reliability is represented by the anticipated failure rate.**

The agglomerated historical data generally agree with the Crow-AMSAA least squares regression, but there are deviations of up to 10%. These deviations remind us of the stochastic nature of the data and non-modeled sources of variation. Variations include year-to-year climatic deviations and solar wind cycles which affect cosmic radiation flux.

Fig. 6 provides a revised view of the forecasted failure rates of Fig. 5. In Fig. 6, the failure rates, expressed as failures per 100 km per "Y" year(s) or "F/cKm/Yyr", are displayed for each cable vintage from 1970 to 2005 at 1 year, 2 years, 4 years, 8 years, and 16 years, where "Y" is the number of years. As in Fig. 5, Fig. 6 assumes that rehabilitation efforts are suspended for all future years. Starting from the bottom line of Fig. 6, each line above represents a doubling of the previous time period. The y-axis is logarithmic. Each doubling of the time *more* than doubles the anticipated failure rate. For example, for 1972 vintage cables the increase in failure rates corresponding to 2X, 4X, 8X, and 16X increases in the time frame are 2.2X, 5.2X, 14.0X, and 48.3X respectively.

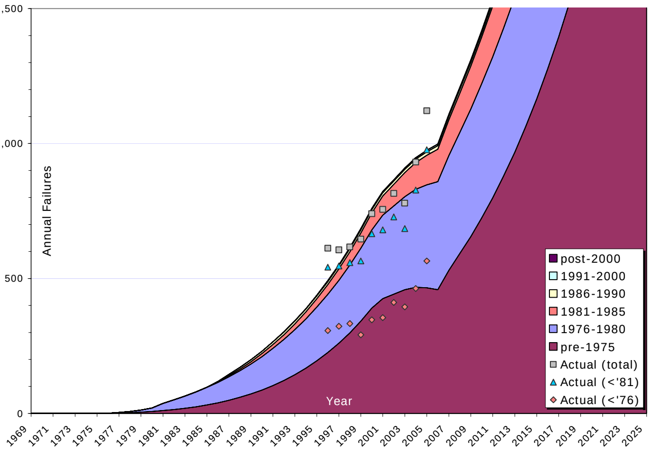

**Figure 5. Agglomerated historical failure data plotted alongside Crow-AMSAA forecast.**

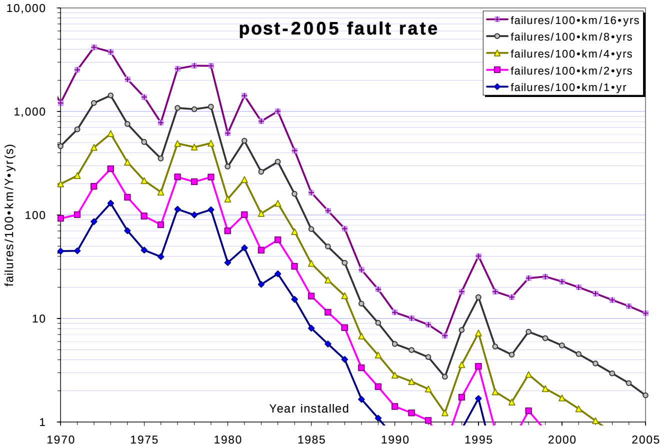

**Figure 6. Fault rate by vintage with no rehabilitation.**

## VII. DIAGNOSTIC CONUNDRUM

Off-line diagnostic testing is inherently destructive. As demonstrated in [3], even the act of de-energizing a circuit creates stress on the cable. Each off-line test then applies some additional electrical stress to the cable. It is beyond the scope of this paper to discuss the degree of damage each test entails.

Even if off-line cable diagnostic testing were not destructive, there are two other factors, which appropriately limit its use. These limitations are illustrated conceptually in Fig. 7. Fig. 7 includes the same model statistics as Fig. 6, except the multi-year lines have been removed, leaving only the failure rate over a single year and the y-axis has been expanded downward a decade to show failure performance for all cable vintages from 1970 to 2005. Fig. 7 is also annotated with two "no-test-horizon" lines. Actual horizons are not simple straight lines. The actual horizons vary with the C-A statistics for each vintage. Each circuit owner must establish their own criteria for the two no-test-horizon lines. For many circuit owners, the values of the economic no-test-horizon are greater than the values of the thermodynamic no-test-horizon over a portion of the left side of the graph. There will always be vintages of cable on the right side of Fig. 7 where the cable failure rate is below the economic no-test-horizon. For these cases diagnostic testing of aged circuits should not be utilized. In Fig. 7 the shaded area represents the area where diagnostic testing may be appropriate. In the balance of this section, the no-test-horizon functions will be refined for an exemplary case.

The establishment of the economic no-test-horizon is largely a matter of economics tempered with an understanding of the failure statistics. The details of deferral economics were presented in detail in [4]. In short, if the number of defects (i.e. incipient failures) is low, a large number of cables must be subjected to destructive testing, which requires spending of scarce operations and maintenance (O&M) resources to identify only a small number of defects. The cost per discovered defect is too high.

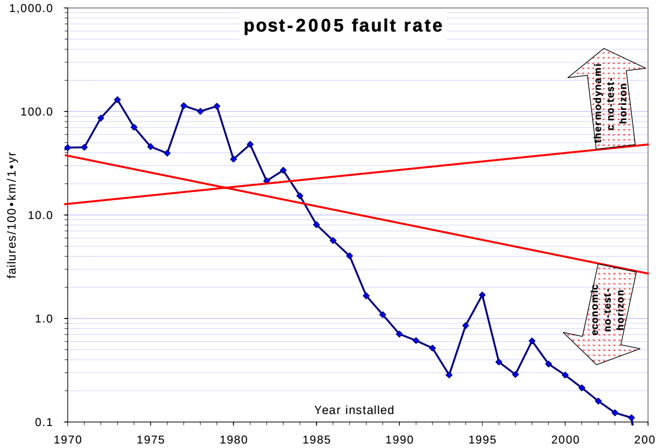

**Figure 7. An illustration of the concept of thermodynamic and economic no-test-horizons for diagnostics.**

Put another way, the total cost to test a cable population must be less than the total cost for other rehabilitation tactics. The total cost to not rehabilitate or ignore cable failures (i.e. allow the cable to fail, repair it, and put it back in service) can be assessed after C-A statistics have been generated. Multiply the number of projected failures over the time period of interest from the C-A analysis by the cost of a single failure and bring all future costs back to present values. Typical costs of failure in North America for direct buried cables compiled by the author are US$3,900 for single phase URD and US$15,400 for three-phase feeder circuits. Costs include:

1. The lost revenue for the outage,
2. the cost of troubleshooting the outage,
3. the cost of repairing or replacing the failed component,
4. the loss of the electrical consumer's good-will,
5. out-of-pocket or insurance costs to cover business interruptions or homeowner damages, and
6. penalties for failing to achieve performance-based rate-making requirements.

To illustrate the calculations, consider the failure rates for 1987 vintage cables for the four future years from Fig. 6. These failure rates (in F/cKm/Yyrs) for Y years 1, 2, 3 and 4 are 4.0, 8.1, 12.3 (not plotted), and 16.6 respectively. The annual failure rates (the differences between consecutive cumulative values) are 4.0, 4.1, 4.2, and 4.3. An example calculation for 100 circuit kilometers (or 1000 100 m cable segments) is presented in Table 3. The net present cost (NPC) to ignore the 1987 vintage cable is US$52,198.

The total cost to apply diagnostic testing to a cable population includes two categories of cost elements, namely the direct cost to test and the consequential costs of testing. The direct cost encompasses the charges from the diagnostic testing supplier, loaded with appropriate management and oversight costs. For this illustration, assume US$4 per cable meter or US$400,000 to test 100 km.

**Table 3. NPC to ignore 100 km of 1987 vintage cable.**

| Assumptions: | | $3,900 | 3% | 11% |
|---|---|---|---|---|
| Year | Failures /cKm/Yyrs | Failure NPC | Inflation factor | discount factor |
| 1 | 4.0 | $14,301 | 1.03 | 0.89 |
| 2 | 4.1 | $13,437 | 1.06 | 0.79 |
| 3 | 4.2 | $12,618 | 1.09 | 0.70 |
| 4 | 4.3 | $11,842 | 1.13 | 0.63 |
| Total | | **$52,198** | | |

The consequential cost element includes two sub-elements. The first sub-element includes the cost of replacing the identified "bad" cables. In [6], values of 20% and greater are suggested. The single datum provided in [6] was 45%. For this example, 20% of the cables tested will be identified as "bad". The statistically anticipated "bad" portion of the cable population is just 16.6 failures or 1.7% of the segment population over a 4-year period. The other 18.3% are either false positives or testing-induced defects. The cost to replace single-phase URD cable is a typical North American value of US$79.54/m ($24.25/ft) or US$7,954/100 m segment.

The second consequential cost sub-element results from testing-induced failures. There will also be other cables, which have testing-induced defects that are not recognized by the diagnostic. These are testing-induced false negatives. In [6] it was reported that 5% of a population of "good" cables, subjected to one kind of test, failed during a subsequent 6-year period. In Table 4 this failure rate is 1% or 10 failures spread evenly over the four years of the analysis.

The net present cost (NPC) to ignore the cable is US$52k; the NPC to diagnose the cable is US$1,686k from Table 4 plus the $400k direct cost. With a factor of about 40X, diagnostic testing cannot be justified for 1987 vintage cable. Fig. 8 extends this analysis to all vintage populations spanning 1970 to 1987. The percentage of cable identified as "bad" was scaled linearly from 20% to 45% based on the C-A estimate of failures over the 4-year period as shown by the dotted line in Fig. 8. The NPC to diagnose cable is higher than the NPC to ignore the cable for all vintages.

**Table 4. Consequential diagnostic costs of 1987 vintage cable. Inflation and discount factor are same as Table 3.**

| | Assumptions | | | |
|---|---|---|---|---|
| | 20% | 1% | $3,900 | $7,954 |
| Year | Identified as "bad" | Testing induced failures | Failure NPC | Replacement NPC |
| 0 | 200 | | | $ 1,590,800 |
| 1 | | 2.5 | $ 8,938 | $ 18,229 |
| 2 | | 2.5 | $ 8,193 | $ 16,710 |
| 3 | | 2.5 | $ 7,511 | $ 15,318 |
| 4 | | 2.5 | $ 6,885 | $ 14,042 |
| | | **Subtotal:** | **$ 31,527** | **$ 1,655,099** |
| | | **Total:** | **$1,686,626** | |

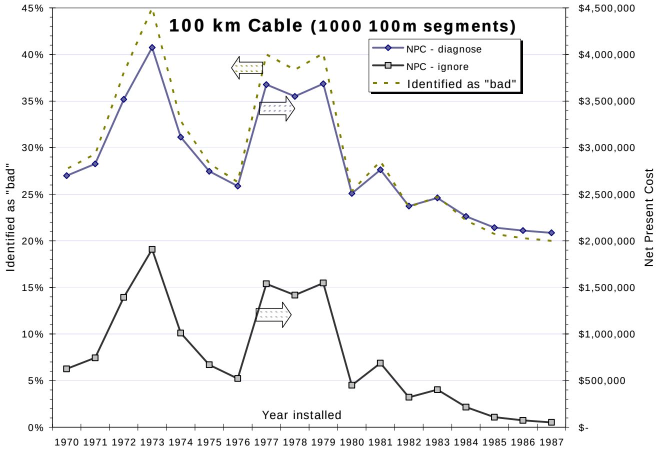

**Figure 8. Economic example of diagnostic testing.**

Ignoring failing cables is not a sustainable tactic as suggested in [1]. Fig. 9 further extends the analysis to include two additional rehabilitation tactics, namely rejuvenate-on-failure and replace-on-failure also described in [1]. Rejuvenation-on-failure is assumed to have an incremental cost of US$3,280 per 100 m cable segment. To simplify the visual comparison, Fig. 9 normalizes the data by showing the ratios of the net present costs (NPC) of each tactic to the NPC of the diagnostic testing approach. For every vintage of cable, the unsustainable ignore tactic and the rejuvenate-on-failure tactic have a NPC ratio of less than 1.0. Diagnostic testing cannot be justified for any cable vintage. Only with the more expensive replace-on-failure tactic are there a handful of cable vintages where the NPC ratio exceeds 1.0 and diagnostic testing meets the economic criterion.

The establishment of the thermodynamic no-test-horizon requires data, which to the author's knowledge, has not been supplied by any diagnostic testing supplier. The missing data would provide insight into the mechanism of the detection of *incipient* failures.

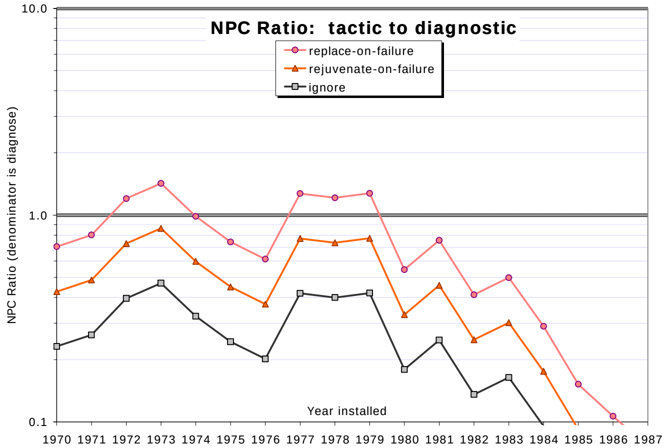

**Figure 9. The ratio of net present costs of several tactical alternatives to diagnostic testing demonstrate testing almost always fails to meet the economic horizon criterion.**

The Merriam-Webster definition of *incipient* is: "beginning to come into being or to become apparent." Combining the principle of the Second Law of thermodynamics in [3] with the industry's knowledge of water tree growth mechanisms and the conversion of water trees to electrical trees, there must be some undetected stage of degradation just prior to incipience. Further, operational aging will convert these pre-incipient failure sites to incipient failure sites. The C-A analysis can predict the frequency of these conversions. At least to some extent, off-line diagnostic tests hasten the conversion of these pre-incipient sites to incipient failure sites. These sites are likely larger water trees. The inability of off-line diagnostic tests to meet the economic criterion is due in no small part to this inescapable truth. On-line diagnostic tests do not suffer this flaw. On-line testing must be repeated often, precisely because they cannot detect today – pre-incipient sites, which may convert tomorrow to incipient failure sites.

The data required to provide a thermodynamic criterion would not be difficult to obtain. The experimenter must divide a reasonably homogenous and statistically significant population of cables into two sub-populations. The diagnostic test can then be applied to one group for off-line testing and to both groups for on-line testing. The failures of both groups should be followed over a multi-year period. Without a well designed control, all results are anecdotes and of little value in making tactical rehabilitation decisions.

For a given population of cables and absent reliable data on diagnostics, some judgment is required to establish the thermodynamic criterion. If the Crow-AMSAA beta is high, the failure rate will accelerate, absent an effort to rehabilitate the cable. Even if a diagnostic test were non-destructive, in practice, removing a handful of incipient failures cannot stop the formation of new incipient failures – only replacement or rejuvenation can accomplish that end.

## VIII. STOCASTIC DIAGNOSTIC

The good news is that a robust statistical analysis of the actual performance of the circuits is the best and least expensive diagnostic of future performance, even if it requires costs for field collection of cable age data. If a circuit owner insists on utilizing destructive diagnostics, starting with a C-A analysis screens out circuits that are not candidates – the very worst and the very best performing cables. Indeed, the often stated purpose of a diagnostic test is to distinguish the "good" cable from the "bad" cable. This purpose is obviated by good record keeping and at a fraction of the cost.

There are three basic items of data required:

1. Vintage cable segment installation records.
2. Cable failure records tied to vintage.
3. Replacement and rejuvenation records.

The circuit owner that provided the data for this paper utilized historical purchasing records and development drawings to compile the item 1 data. An alternative method is to perform a field survey. *Do not let the perfect, be the enemy of the good.* A field survey does not have to include 100% inspection. A modest sample of a development can often establish the age for the entire development. A transformer date is not likely to be too different than the date of cable manufacture and installation. A well designed survey program should cost less than US$0.50 per circuit meter (<15¢/ft).

Every circuit owner should have a comprehensive failure reporting system in place. Aggregate failures are better than no failure data, but failure data that are tied to the vintage of installation dramatically improve statistical confidence and facilitates the fine tuning of strategic options. Failures of rejuvenated cables and treatment dates are available from service suppliers.

Each time a cable is rehabilitated, its participation in its original population is suspended and the rehabilitated circuit begins a new life in a new population with its own set of reliability statistics. Replacement is particularly easy, because the data gathering is a simple extension of item 1. The circuit owner should make provisions in its property records to record the chemical rejuvenation of cables, so that data is not lost to posterity.

## IX. SUMMARY

Cable failures are a stochastic process driven by the most fundamental of principles – the immutable Second Law of thermodynamics. The reliability of non-rehabilitated aging cable can go only one way – down. An inexpensive device to determine which cables are going to fail next has yet to be commercialized. Confounded first by economics and second by the physics and chemistry of ageing cable, off-line diagnostic devices have demonstrated only that they can accelerate the failure of the cable to which they propose salvation. For pennies a meter, the tools of statistics can be applied to the problem. With the foundation of data to which every circuit owner has access, optimum resource allocations can be made. The author will provide a roadmap describing the mathematics of strategic resource optimization in a future paper.

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